Russian Academy of Sciences
Program Systems Institute

System Analysis Research Center

Mathematical Models and Optimal Processes in Macrosystems (Thermodynamics and Microeconomics)

The main research field of SARC is the investigation of optimal processes and extreme performance of macrosystems. Results of this investigation are applied to irreversible thermodynamics and microeconomics. Macrosystems consist of a large amount of individually uncontrolled particles (e.g. moleculae in thermodynamics, economic agents in economics, etc.). Control of the macrosystems is possible at a macrolevel. It means that one can have an influence on a whole set of particles.

Main Problems

1. Quantitative measures of irreversibility.
2. Minimal dissipation processes.
3. Stationary state of a system with an active subsystem.
4. Extreme performance of the active subsystem in closed, open and nonstationary macrosystems.
5. Realizability area of macrosystems.

Quantitative measure of irreversibility of processes is one of the characteristics of macrosystems. The value of its change determines the resources quantities required to return the system to its initial state.

Entropy is such a measure in thermodynamics. Wealth function introduced in microeconomics serves as a measure of irreversibility. During any process the irreversuibility measure of a system can not decrease. Its increment rate is called *'dissipation'*.

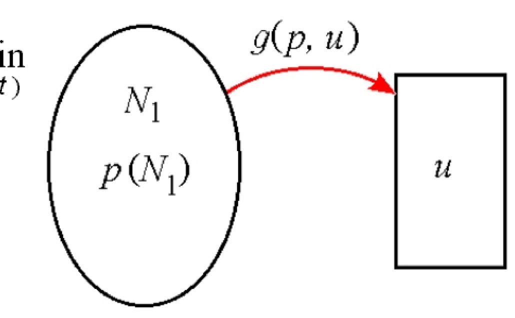
Minimal Dissipation Processes

Thermodynamics

$$\bar\sigma = \frac{1}{\tau}\int_0^\tau g(p,u)X(p,u)dt \to \min_{u(t)}$$

$$\frac{1}{\tau}\int_0^\tau g(p,u)dt = \bar g$$

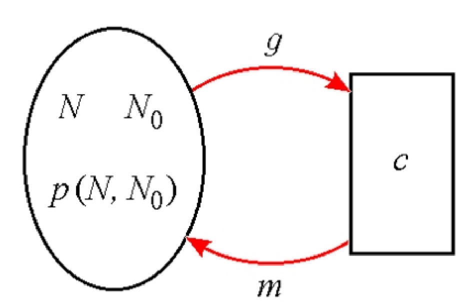
$$\dot N_1 = -g \;\Rightarrow\; \dot p = \varphi(p,u), \quad p(0)=p_0, \quad \varphi \neq 0\,(p \neq u)$$



For the case $\varphi = \alpha(p)g(p,u)$

$$\frac{g^2}{\partial g/\partial u}\frac{\partial X}{\partial u} = \text{const}$$

Microeconomics



$$N_0(\tau) \to \min_{c(t)}$$

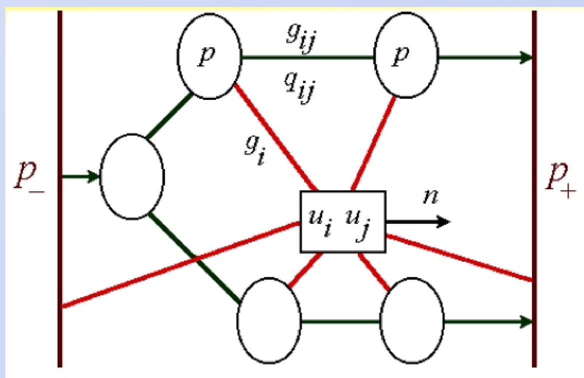
$$\frac{dN_0}{dt} = cg(c,p), \quad N_0(0) = N_0^0,$$

$$\frac{dN}{dt} = -g(c,p), \quad N(0) = N^0,$$

$$\frac{1}{\tau}\int_0^\tau g(c,p)dt = \bar g.$$

$$\frac{d}{dN}\left[\frac{\partial g/\partial c}{g^2}\right] = \frac{(\partial g/\partial p)(\partial p/\partial N_0)}{g^2}$$

Stationary State of an Open System with an Intermediator



Thermodynamics

n – Power $\quad p_{1i}$~T_i
q – Heat flux
g – Substance flux
p – Intensive variables

$$n = \sum_i [q_i(p_i,u_i) + g_i(p_i,u_i)] \to \max_u$$

Subject to $\sum_j q_{ij}(p_i,p_j) = q_i, \quad \sum_j g_{ij}(p_i,p_j) = g_i, \quad \sum_i g_i = 0,$

$$\sum_j \left[g_{ij}s_{ij} + \frac{q_{ij}}{p_{1j}}\right] = 0, \quad i = 1,\ldots,m \qquad \sum_i\left[g_i s_i + \frac{q_i}{u_{1i}}\right] = 0.$$

If $g = 0,\ q_{ij} = \alpha_{ij}(T_i - T_j)$, то

$$\sum_i \alpha_i \frac{T_i}{u_i} = \sum_i \alpha_i; \quad u_i^2 = \Lambda\frac{T_i}{1-\lambda_i}$$

$$\alpha_i\left(1 + \frac{\Lambda}{u_i} - \lambda_i\right) = \lambda_i\sum_j \alpha_{ij}, \quad i = 1,\ldots,m$$

If $m = 2,\ T_1 = T_+,\ T_2 = T_-$, то

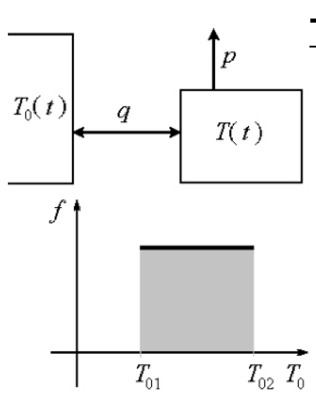
$$u_1^* = k\sqrt{T_+}, \quad u_2^* = k\sqrt{T_-}, \quad \eta = 1 - \sqrt{\frac{T_-}{T_+}},$$

$$N_{\max} = \frac{\alpha_1\alpha_2}{\alpha_1+\alpha_2}\left(\sqrt{T_+} - \sqrt{T_-}\right)^2$$ – Maximal power of a heat engine

If $g=AX$ (where A is Onsager matrix) then Prigogine's extremal principle is valid for any u.

Nonstationary Reservoirs

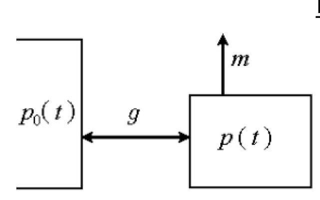
If reservoirs are nonstationary it is possible to obtain power output while the working fluid contacts with the only reservoir. Intensity of the heat flux depends on current temperature of the reservoir. In microeconomics profit of a firm can be extracted during operation at a single market.



Thermodynamics $\quad \bar p = \overline{q(T_0(t),T)} \to \max_{T(t)}$

$$\frac{1}{T^2}\frac{q}{\partial q/\partial T} - \frac{1}{T} = \text{const}$$

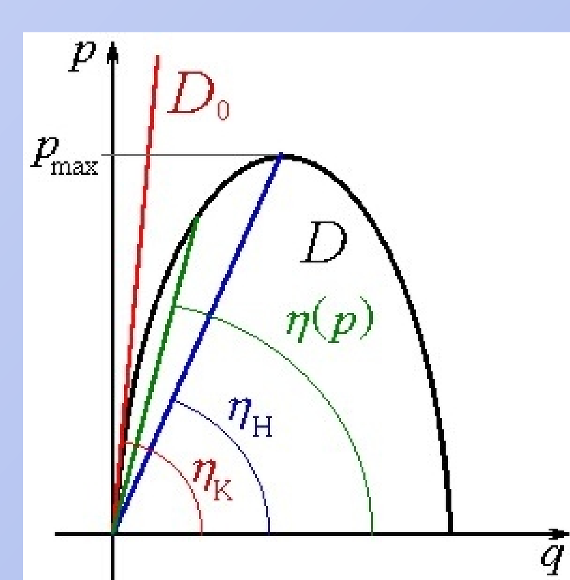
$$\bar p_{\max} = \alpha\left[\frac{T_{01}+T_{02}}{2} - \frac{4}{9}\left(\frac{T_{02}^{3/2} - T_{01}^{3/2}}{T_{02} - T_{01}}\right)^2\right]$$



Microeconomics $\quad \bar m = \overline{pg(p_0,p)} \to \max_{p(t)}$

$$\frac{g}{\partial g/\partial p} + p = \frac{\int_0^\tau \frac{\partial q}{\partial p}p\,dt}{\int_0^\tau \frac{\partial q}{\partial p}dt}$$

Realizability Area



$$\eta_K = 1 - \frac{T_-}{T_+}$$

$$\eta_H = 1 - \sqrt{\frac{T_-}{T_+}}$$

$$\sigma = 0\,(\tau\to\infty, p\to 0) \sim D_0 \qquad D \subset D_0$$
$$\sigma(p) > 0\,(\tau, p > 0) \sim D$$

$$\eta(p) = \frac{p}{q} \le \frac{1}{2}\left(\frac{p}{\bar{\alpha} T_+} + \eta_K\right) + \sqrt{\frac{1}{4}\left(\frac{p}{\bar{\alpha} T_+} + \eta_K\right)^2 - \frac{p}{\bar{\alpha} T_+}}$$

A set of possible states of a macrosystem is restricted not only by constraints peculiar to concrete problems. Macrosystems are characterized by a factor of irreversibility. This factor must increase in close systems. In open macrosystems dissipation of energy and capital are nonnegative. These restrictions bound the realizability area.

A general method of determination of realizability area for closed macrosystems with an active subsystem is the following:

1. To write balance equations including balance relation for irreversibility factor S.
2. To find a minimal value $\sigma = \sigma_{\min}$ corresponding to a pregiven state of the system subject to restrictions on duration of the process. This value characterizes a process of minimal dissipation.
3. The balance equations at $\sigma > \sigma_{\min}$ determine realizability area D.

Fields of research work of the System Analysis Research Center

- Optimal Control Methods for Irreversible Thermodynamic Systems and Estimations of Extreme Performance of these Systems
- Mathematical Models and Optimal Processes in Irreversible Microeconomics
- Optimal Control of Temperature Fields and Energy Saving Problems (Cooling in Supercomputers, Energy Saving in House-Building)
- Problem on Equivalence of Differential Equations
- Geometrical Conditions of Solvability of Convolution Equations

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